

# Interplay between charge & pairing modulations in cuprate high- $T_c$ superconductors

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Didier Poilblanc\*



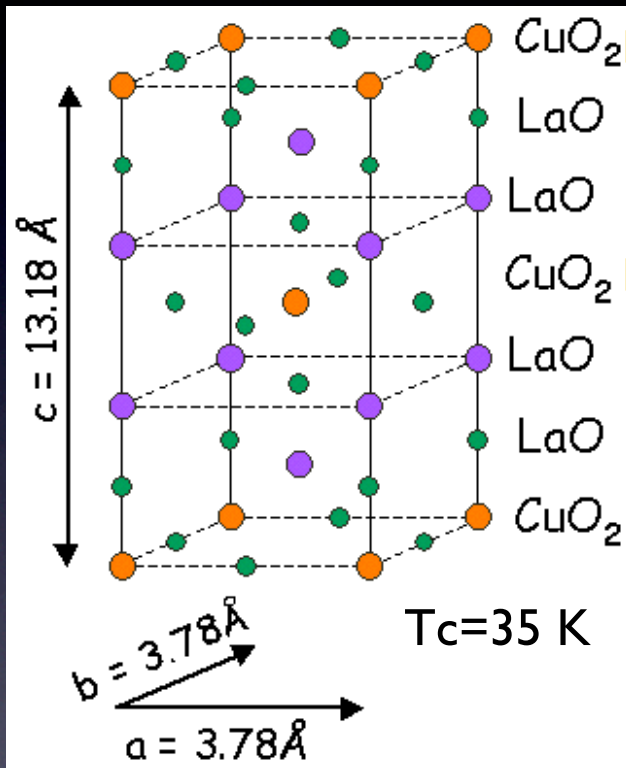
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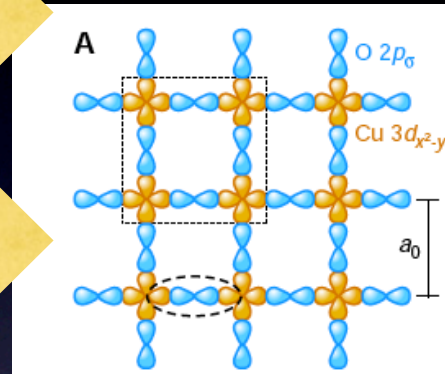
# Outline

- Motivations: experimental observation of charge ordering in High- $T_c$  materials
- Theoretical framework: t-J model and variational wavefunctions
- Results on superconducting RVB hole stripes

# The Cuprates



Layered structure with  $\text{CuO}_2$  planes



Cu d-orbitals:  
small overlap  
strong correlation

2D square lattice

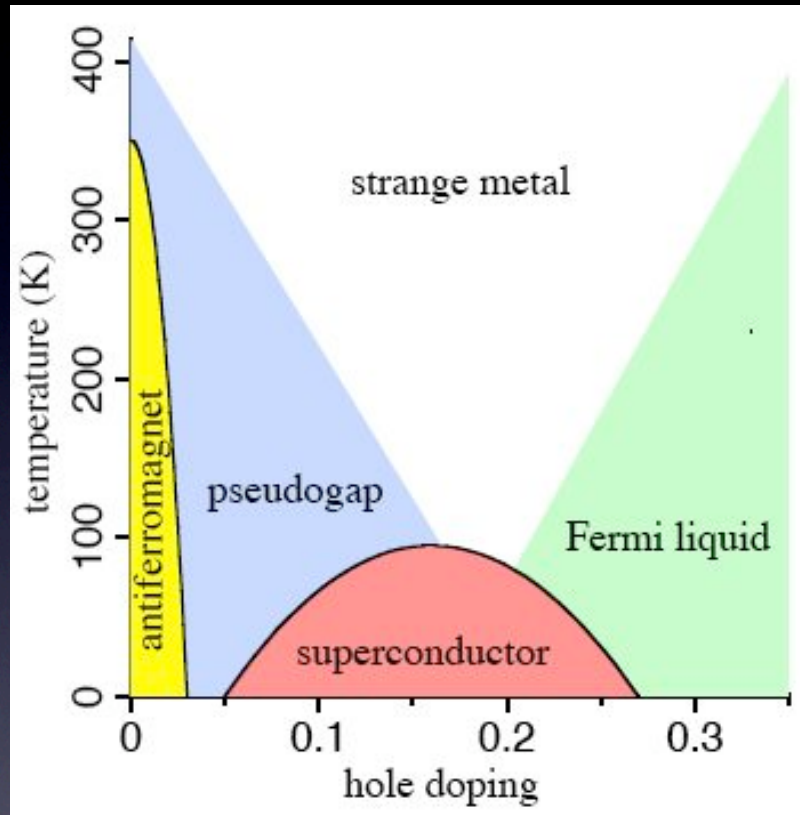
Interesting Physics upon doping

\*Undoped:  $\text{La}_2\text{CuO}_4$ : 1 electron per site

\*Doped:  $\text{La}^{3+}$  substituted with  $(\text{Ba}, \text{Sr})^{2+}$   
introduction of extra carriers (holes)  
in the planes



# High-Tc Phase diagram

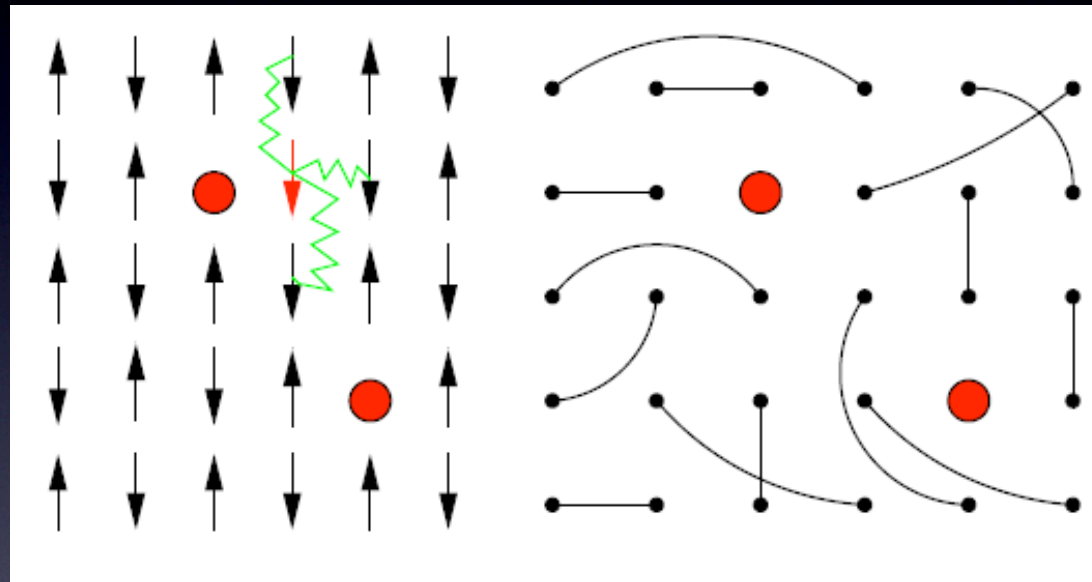


- \*  $S=1/2$  Mott insulator (AF)
- \* Pseudogap for underdoped and Non-Fermi liquid
- \* d-wave superconductivity
- \* Fermi liquid for overdoped

Strong correlation is ubiquitous in High-Tc  
Superconducting state emerges from  
doping a Mott insulator

# The RVB scenario upon doping

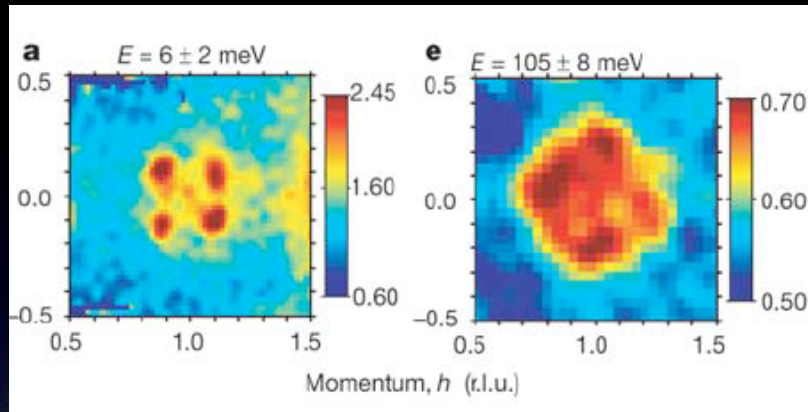
Holes frustrate antiferromagnetism



The RVB state regains the lost AF exchange  
by the resonance between many different configurations

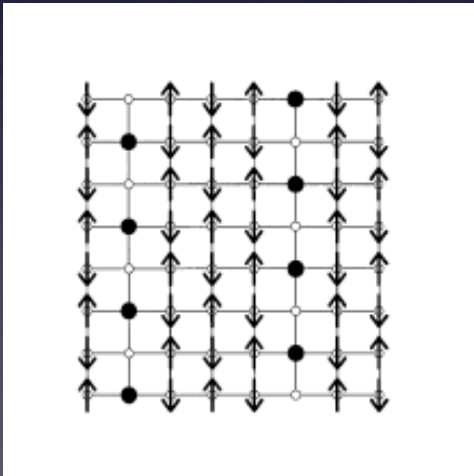
The RVB state naturally becomes a superconductor  
since the pairing already exists

# Neutron scattering: AF Stripes



$\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$   
at doping  $x=1/8$

[Tranquada et al. Nature 1995]



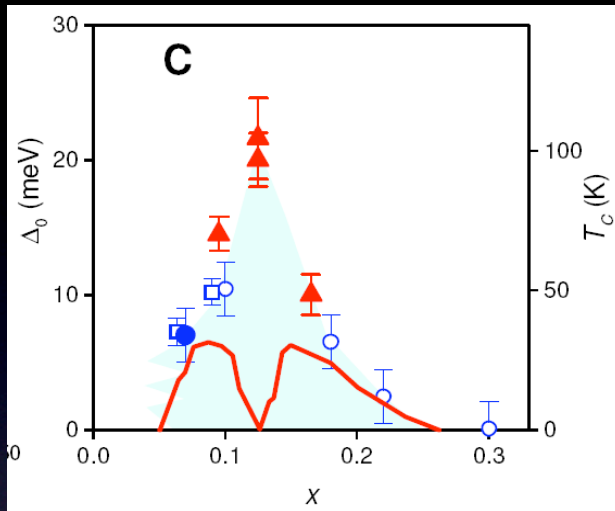
## ANTIFERROMAGNETIC STRIPE SCENARIO

Spatially ordered state with holes concentrated *unidirectionally* between AF domains

Zaanen et al. PRB 89  
Poilblanc-Rice PRB 89

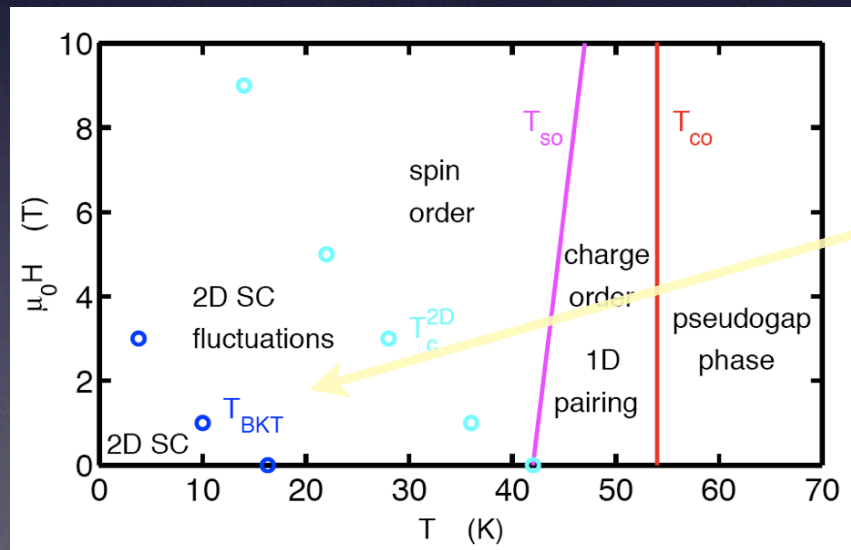


# Stripes are compatible with pairing !



LaBaCuO:  $T_c \sim 0$  at doping  $x = 1/8$   
but (ARPES) d-wave gap still  
there!

[Valla et al., Science 2006]



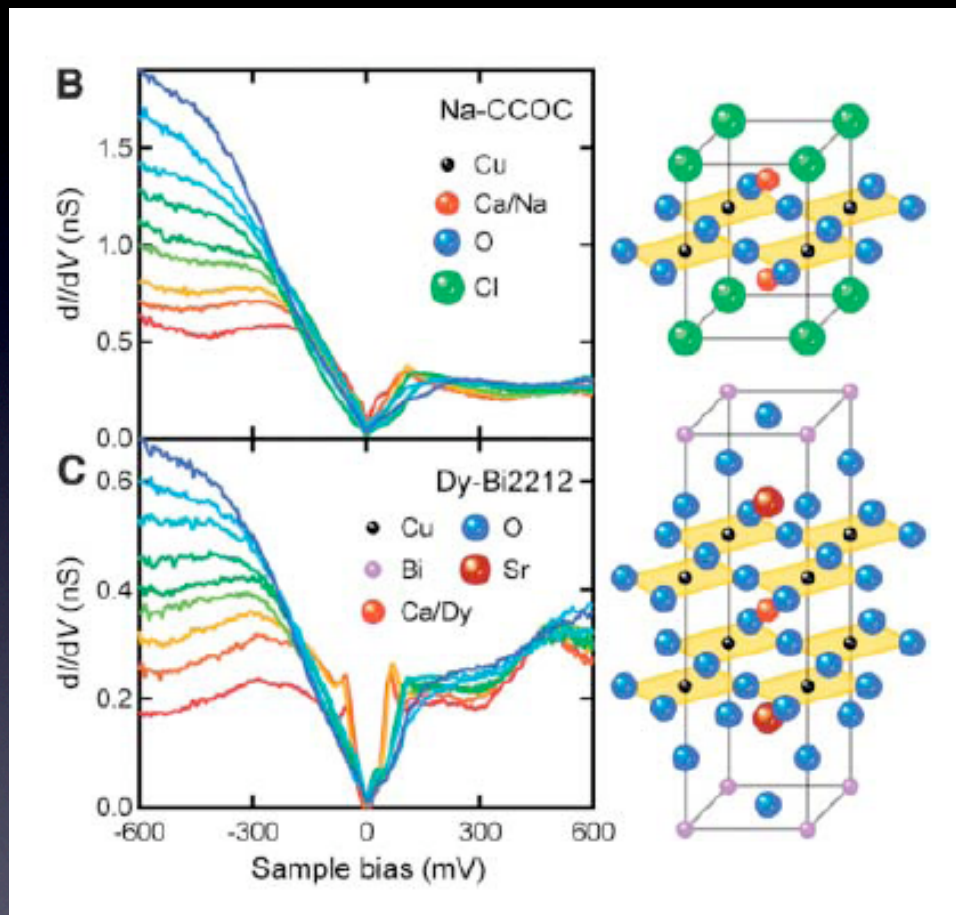
Planes are (Josephson) decoupled  
but pairing exists!

[Berg et al., PRL 2007]

How can one get 2D SC ?

[Li et al., PRL 2007]

# STM Experiments:



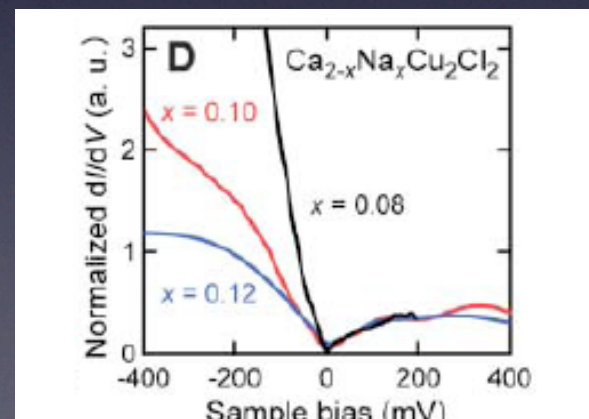
Large spatial dependence of the Tunnelling asymmetry

DIFFERENTIAL CURRENT

$$dI/dV(r,V) = f(r,z) N(r,E=eV)$$

\* $N(r,E)$  LOCAL DOS

\* $f(r,z)$  tunnelling matrix element (unknown)



EXTRACT  $e^-$

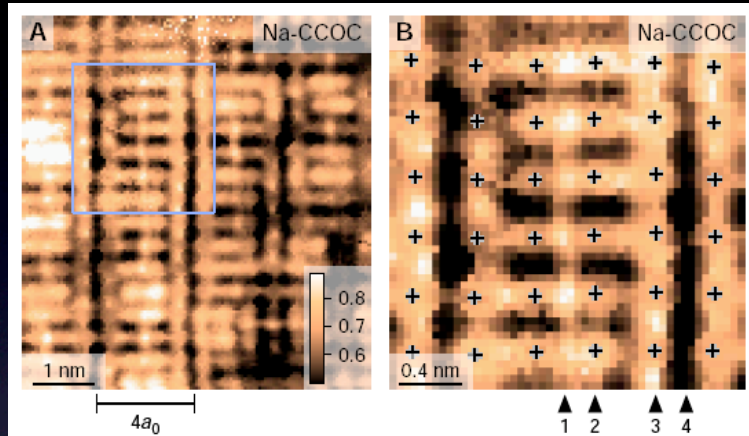
INSERT  $e^-$



# STM-experiments: R-maps

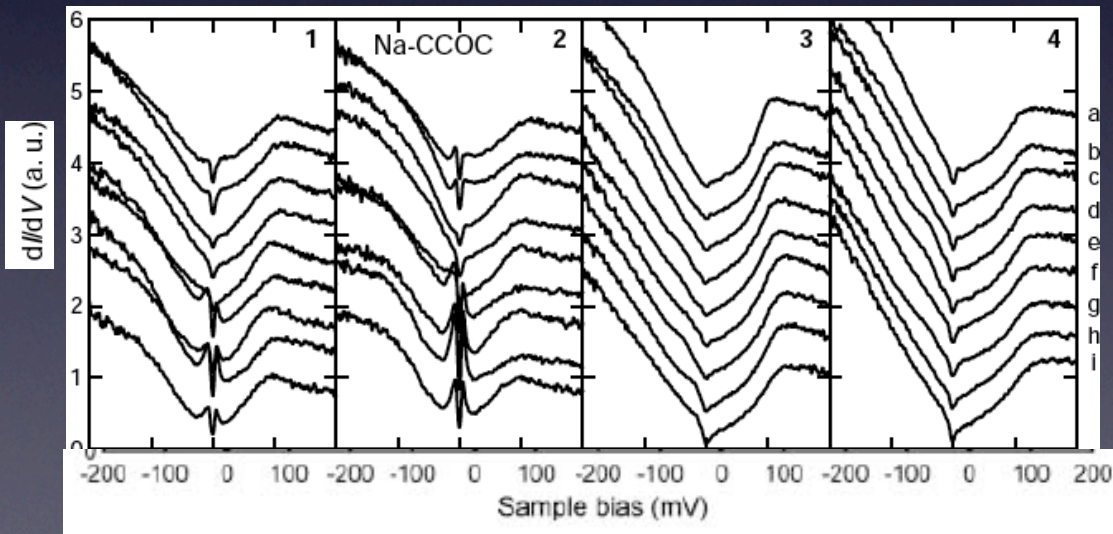
$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$  and Dy-Bi2212 (at  $T < T_c$ )

[Kohsaka et al. Science 2007]



$$R(r, z, V) = \frac{I(r, z, +V)}{I(r, z, -V)} \sim \frac{x(r)}{1 - x(r)}$$

$R \approx 1 \Rightarrow$  more holes  
 $R < 1 \Rightarrow$  less holes



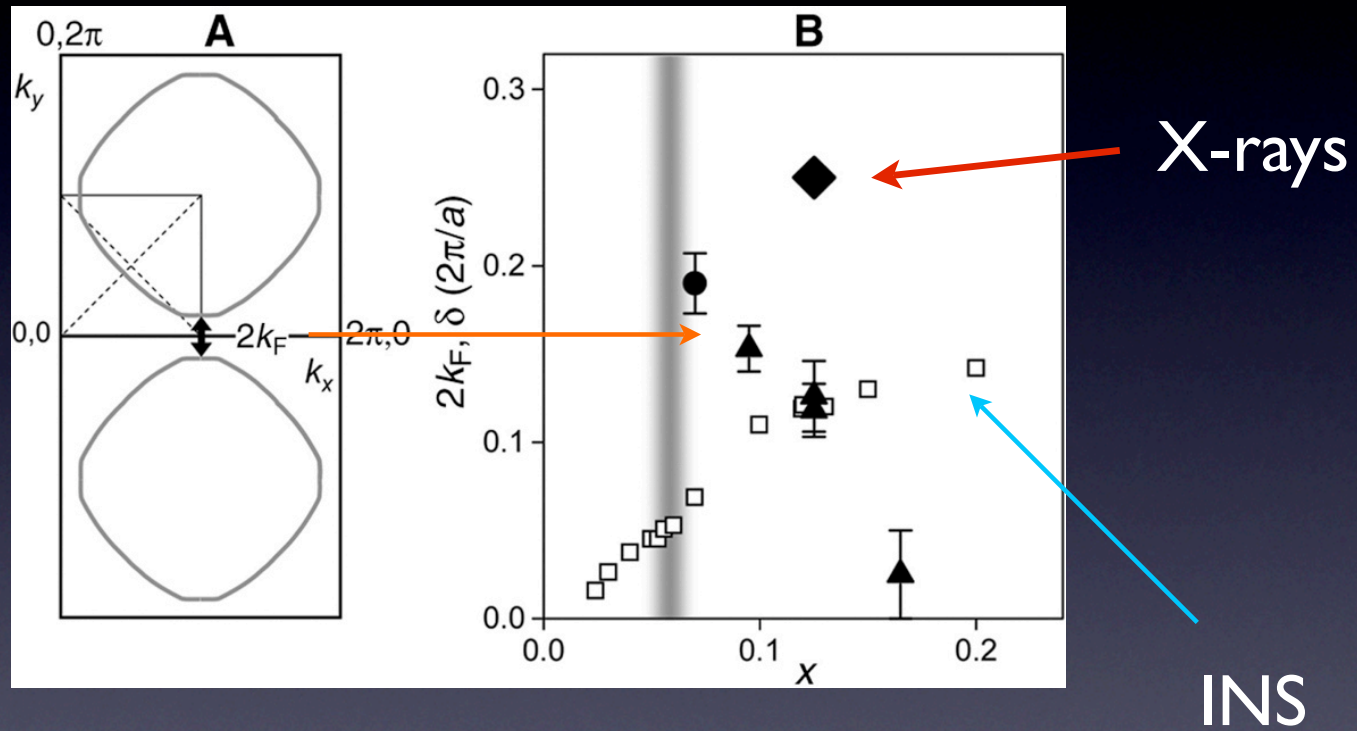
HOLE RICH

HOLE POOR

Bond-centered  
 unidirectional  
 patterns

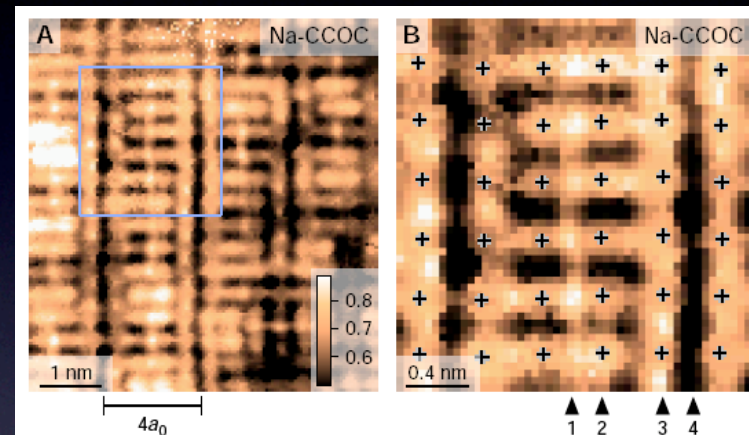
Different low-energy  
 properties

# Not a Fermi surface nesting mechanism !



[Valla et al., Science 2006]

# SPATIAL ORDER + SUPERCONDUCTIVITY

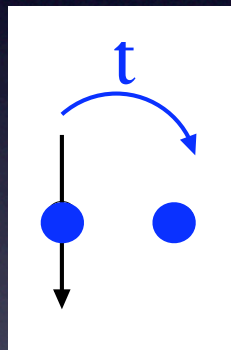


GOAL: describe **superconducting hole-stripes**  
within RVB framework ?



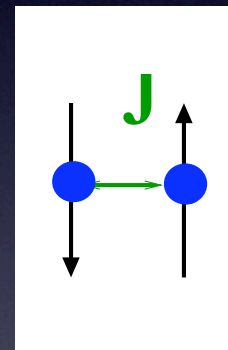
# The t-J model

$$H_{tJ} = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + h.c. + J \sum_{\langle ij \rangle} S_i \cdot S_j$$

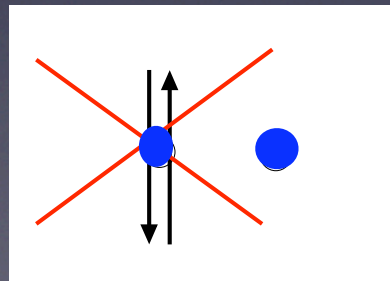


KINETIC  
TERM

+



AF  
EXCHANGE  
TERM



CONSTRAINT of  
NO DOUBLE OCCUPANCIES

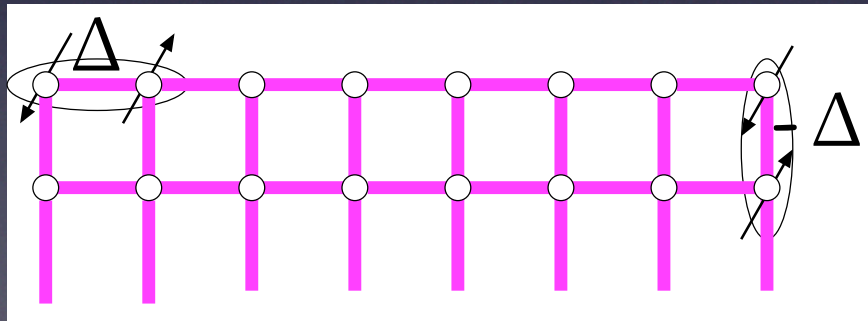
# RVB variational state

$$H_{BCS} = H_{kin} + \sum_{ij} \Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger + \mu \sum_i n_i + h.c.$$

Uncorrelated state  $|D\rangle$

$$|\Psi_{RVB}\rangle = \prod_i (1 - n_{i\uparrow} n_{i\downarrow}) |D\rangle$$

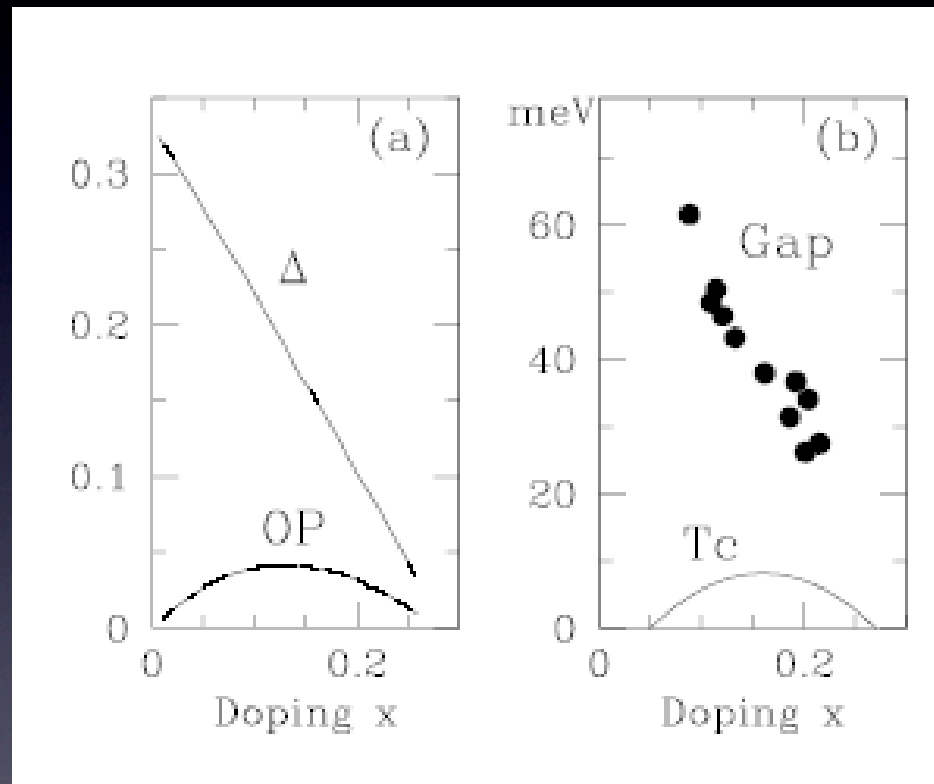
Strongly correlated wavefunction



$\Delta, \mu$  are variational parameters

All  $\Delta_{ij}$  uniform with d-wave symmetry

RVB => Correct behavior of  
pseudo-gap & SC order parameter

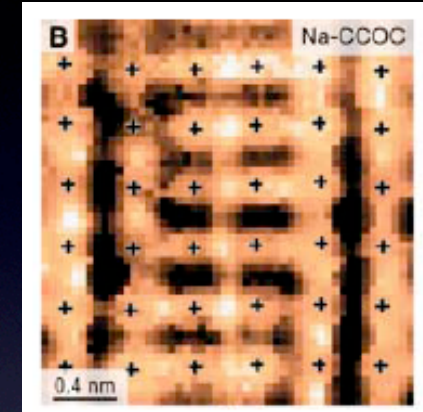
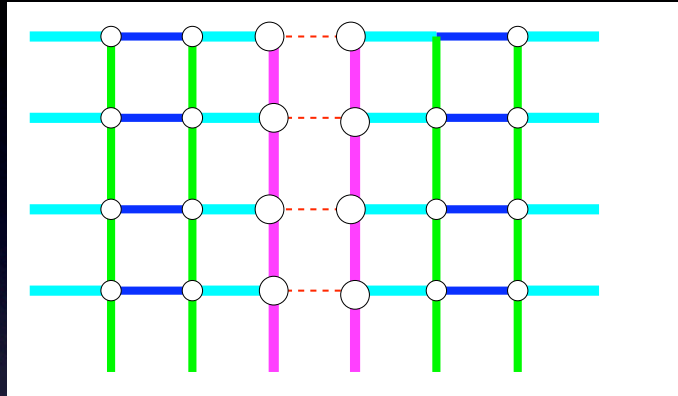


Anderson et al. J.Phys. C 2004

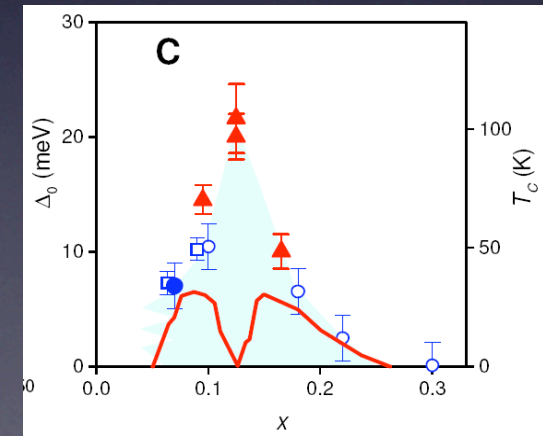
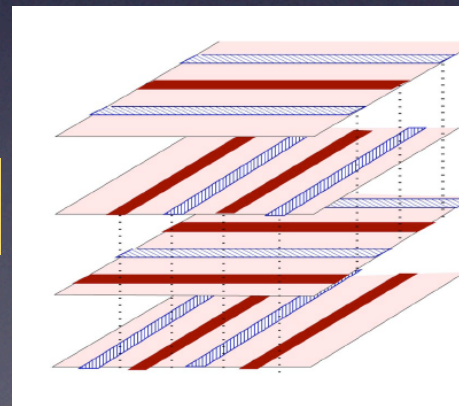
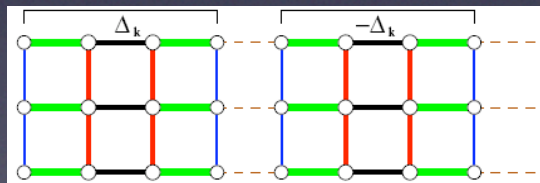


# Modulated RVB state ?

\* superconducting RVB hole stripes



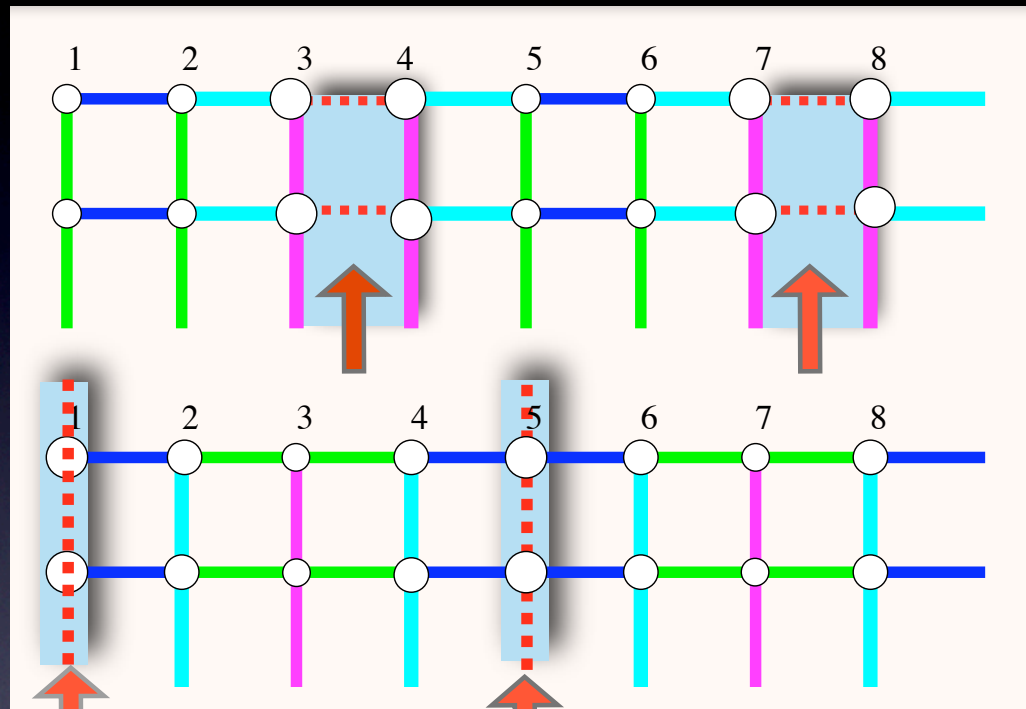
\*  $\pi$ -shift RVB hole stripes



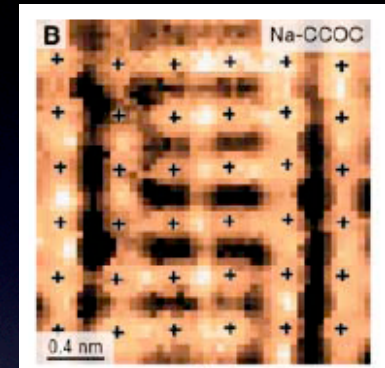
# Superconducting stripes

We allow for inhomogeneous  $\Delta_{ij}$

Bond  
centered

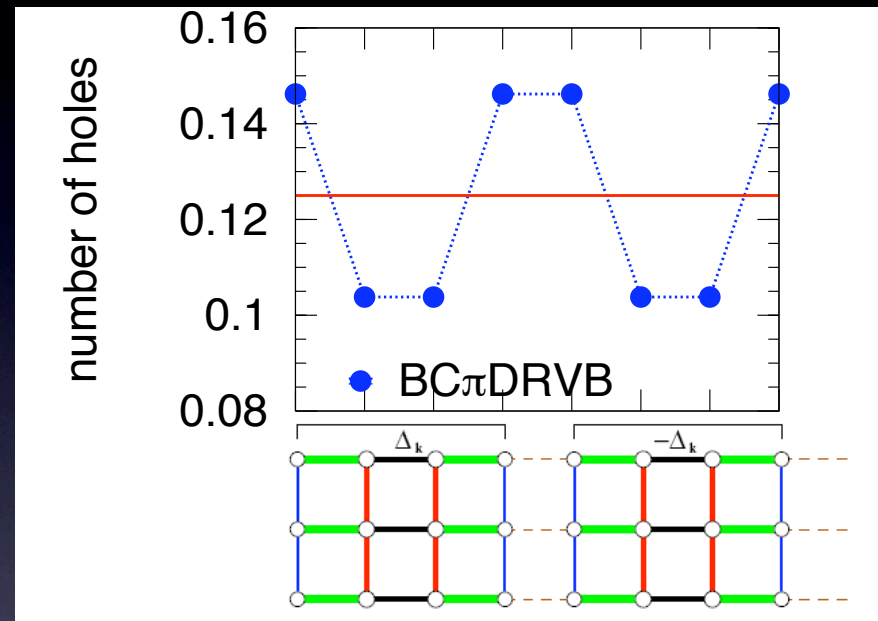


Site  
centered



Create line-defects in the RVB state  
Impose  $\Delta_{ij}=0$  along one direction,  
with periodicity  $1/2x$

# Computed charge modulation



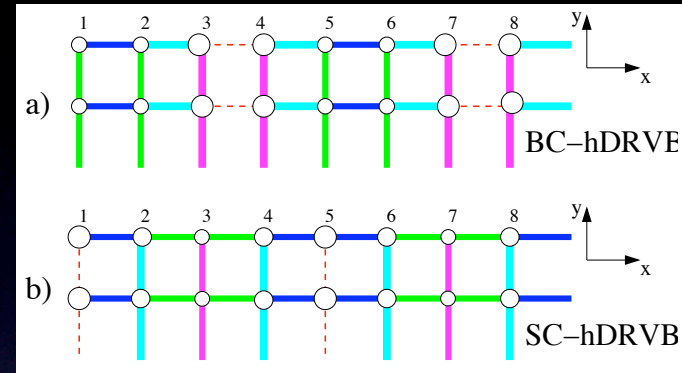
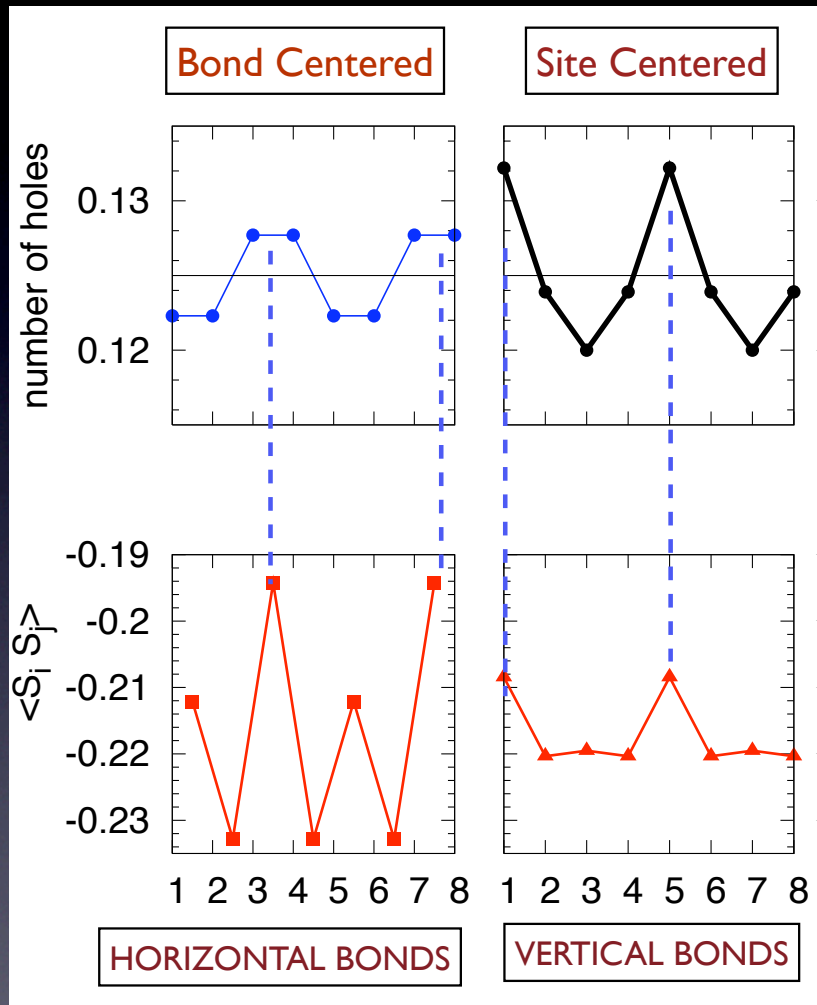
$\tau/J=3$ ,  
doping 1/8  
up to  
16x16 clusters

2 NON-EQUIVALENT SITES

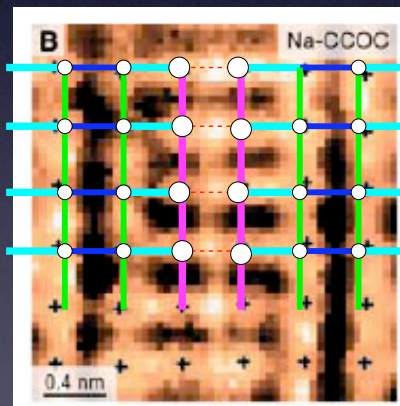
Holes concentrate where  
spin-pairing is smaller



# Bond vs Site centered



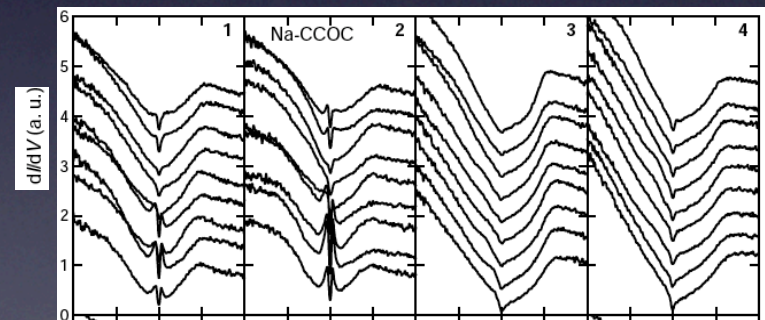
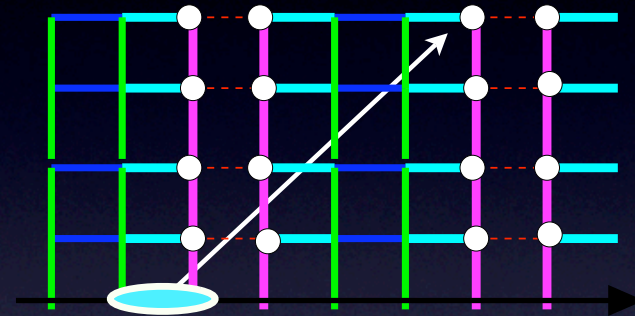
Hole stripes emerge



# Superconductivity is modulated !

$$P_s^2(r) = \langle \tilde{\Delta}_{s+r}^\dagger \tilde{\Delta}_s \rangle$$

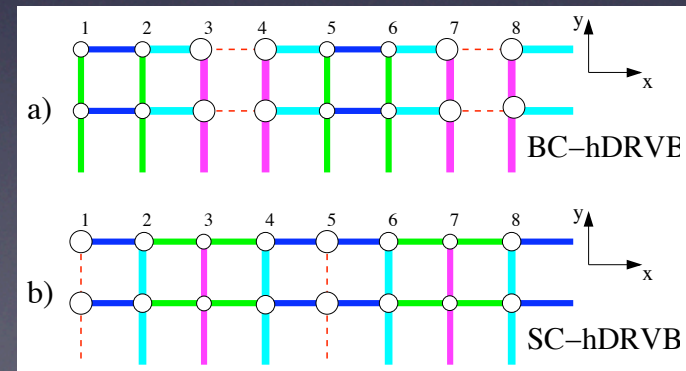
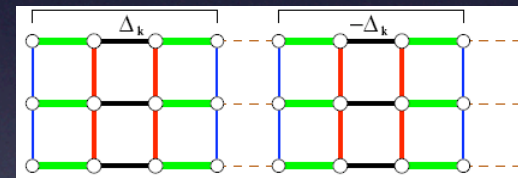
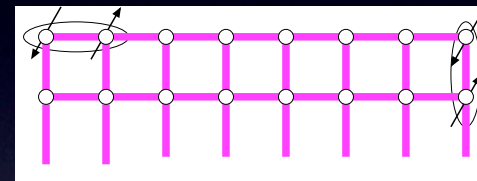
$$\tilde{\Delta}_s^\dagger = c_{s\uparrow}^\dagger c_{s+a\downarrow}^\dagger - c_{s\downarrow}^\dagger c_{s+a\uparrow}^\dagger$$



# Energies are really close ( $\sim 10^{-4} t$ )

$t/J=3$ ,  
doping  $1/8$   
up to  
 $16 \times 16$  clusters

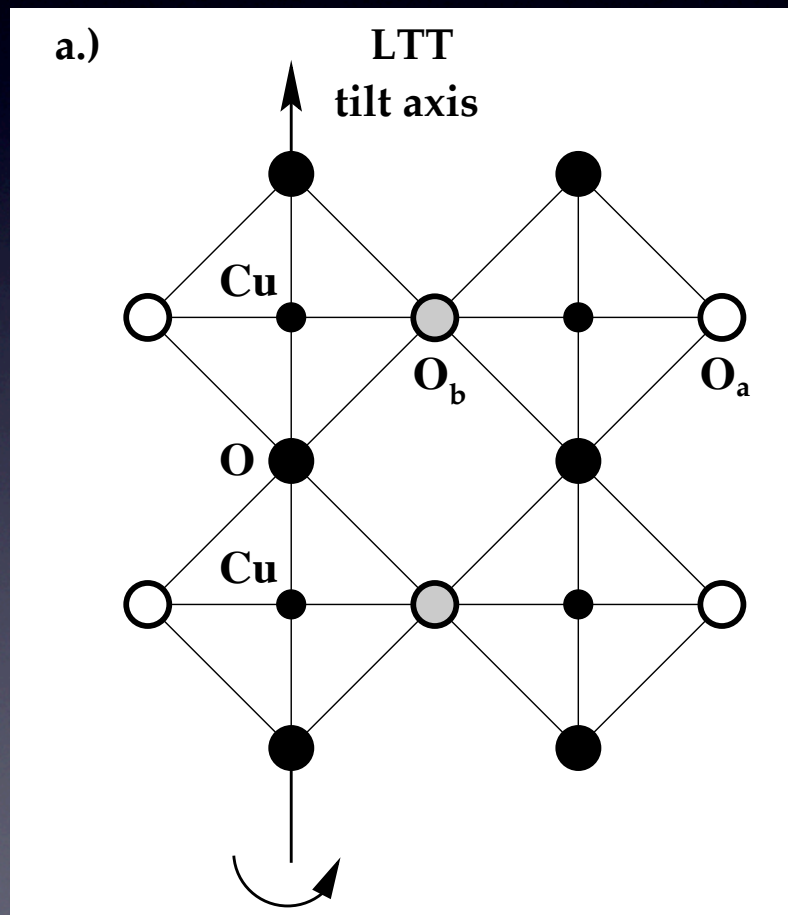
WF	$E_{VMC} [t]$
RVB	-0.45564(3)
SFP	-0.44630(3)
pi-DRVB	-0.44529(3)
BC-hDRVB	-0.45490(3)
SC-hDRVB	-0.45530(3)





# Lattice distortion

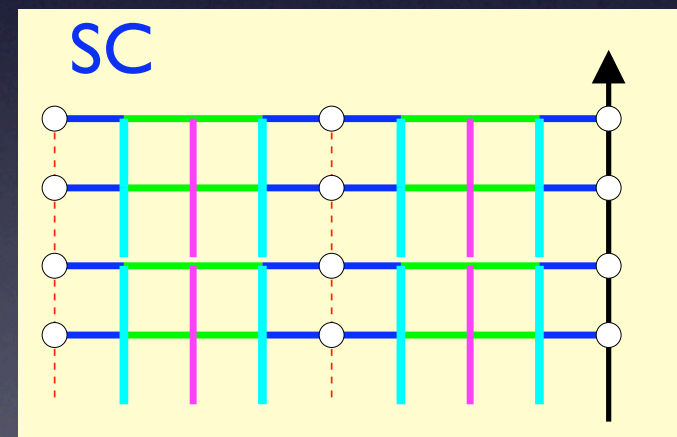
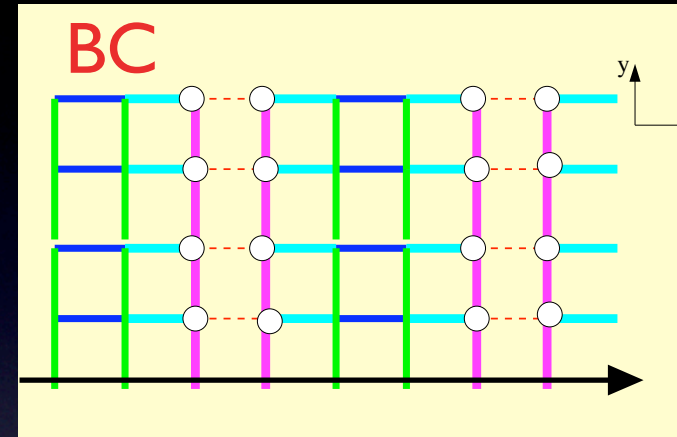
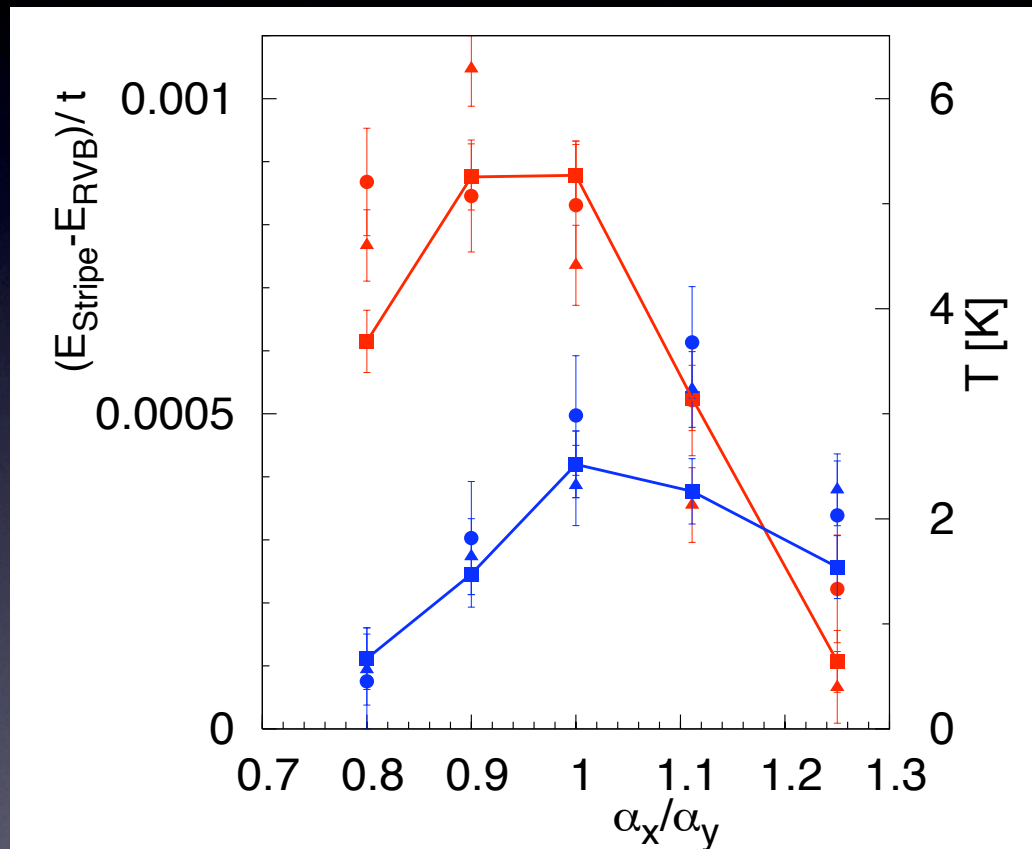
$$H_{tJ}^{\alpha} = -t \sum_{\langle ij \rangle, \sigma} \alpha_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. + J \sum_{\langle ij \rangle} \alpha_{ij}^2 S_i \cdot S_j$$



The tilt in the oxygen octahedra induces a different  $t$  and  $J$  along  $x$  and  $y$ :

tilt axis along  $y$ :  
 $\alpha_x < 1$  and  $\alpha_y = 1$

# Lattice LTT distortion further stabilizes the superconducting stripes



Tilt axis along y

Tilt axis along x

# Other related work

- \* Himeda, Kato & Ogata, PRL 2002  
[ simple cosine modulation of SC ]
- \* Berg, Fradkin, Kim, Kivelson, Oganessian,  
Tranquada & Zhang, PRL (2007)  
[Dynamical layer decoupling scheme]
- \* Yang, Chen, Rice, Sigrist and Zhang, arXiv:0807.3789  
[Mean-field RVB including spin ordering]



# Conclusions

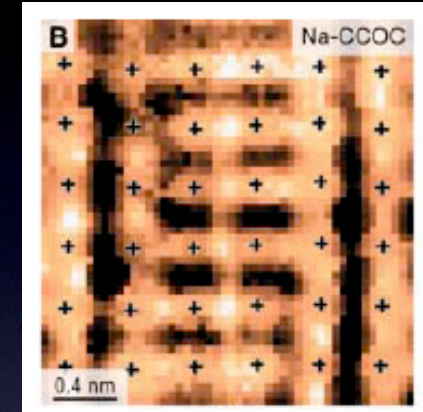
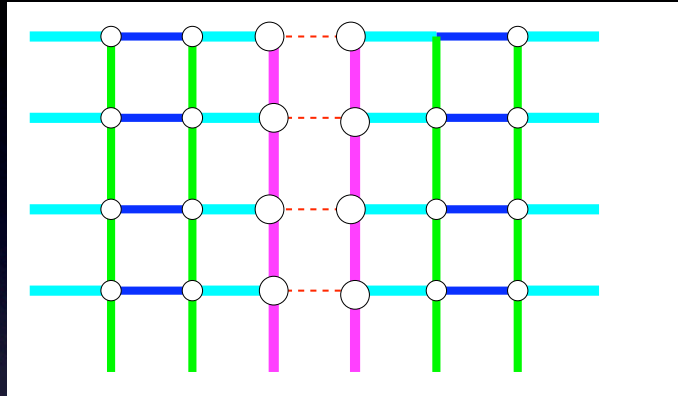
- Evidence that modulated superconducting states are energetically competitive with the uniform RVB.
- Holes patterns/superconducting regions form along unidirectional domains
- Lattice distortion further stabilizes superconducting stripes
- Impurities might also ...

## References:

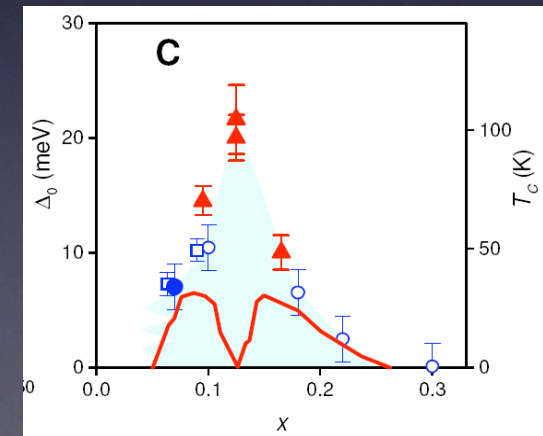
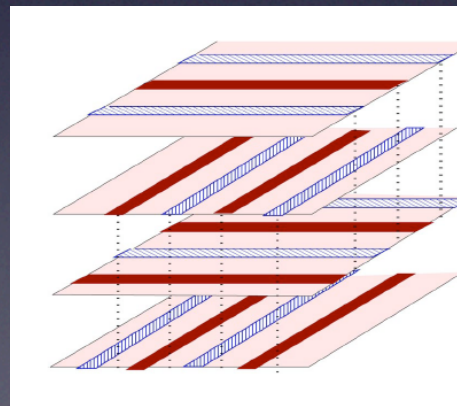
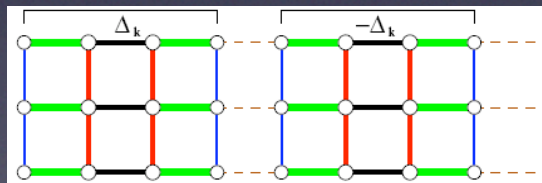
- (I) Raczkowski et al., PRB (RC) **76**, 140505 (2007)
- (II) Capello et al., PRB **77**, 224502 (2008)
- (III) Capello and Poilblanc, PRB **79**, 224507(2009)

# Partial summary

\* superconducting RVB hole stripes



\*  $\pi$ -shift RVB hole stripes



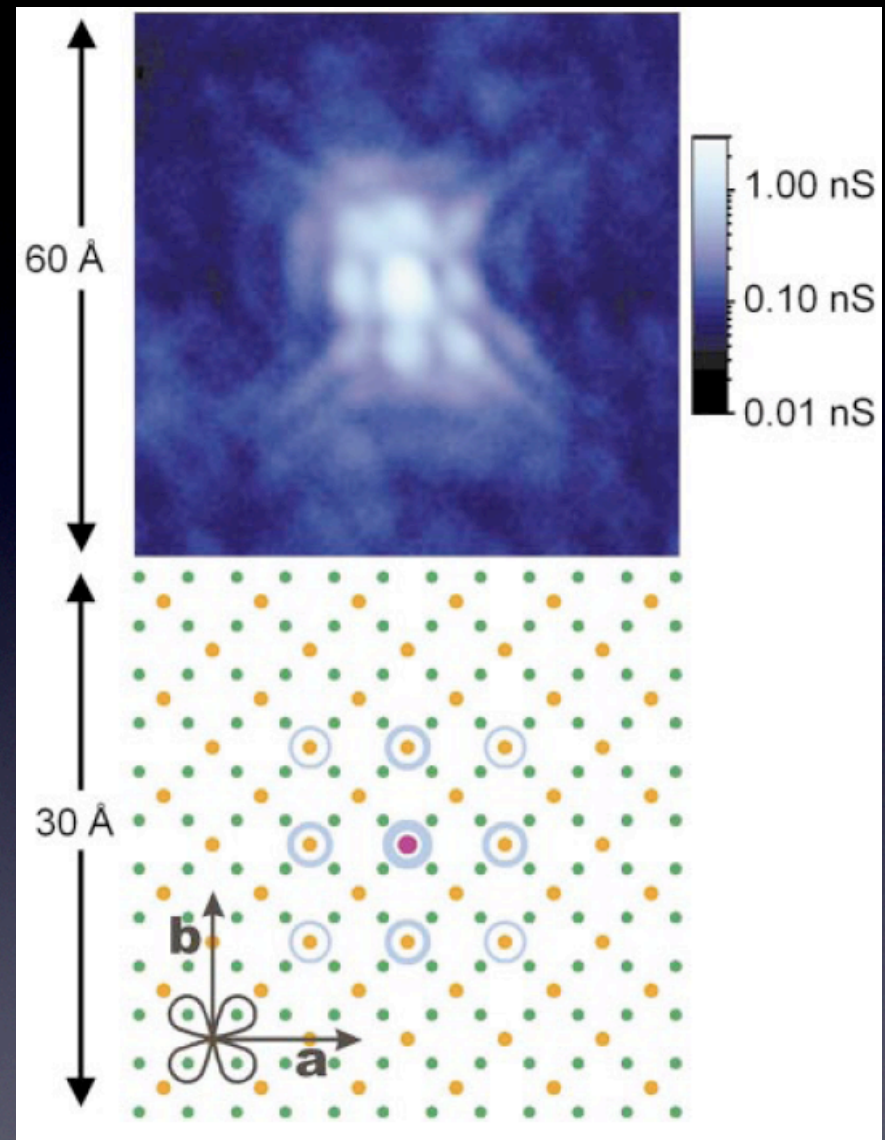


# Role of impurities ?

Low-energy DOS around  
Zinc impurity in SC state

- Large DOS along nodal directions
- Suppression of SC within  $\sim 15\text{\AA}$  from Zn

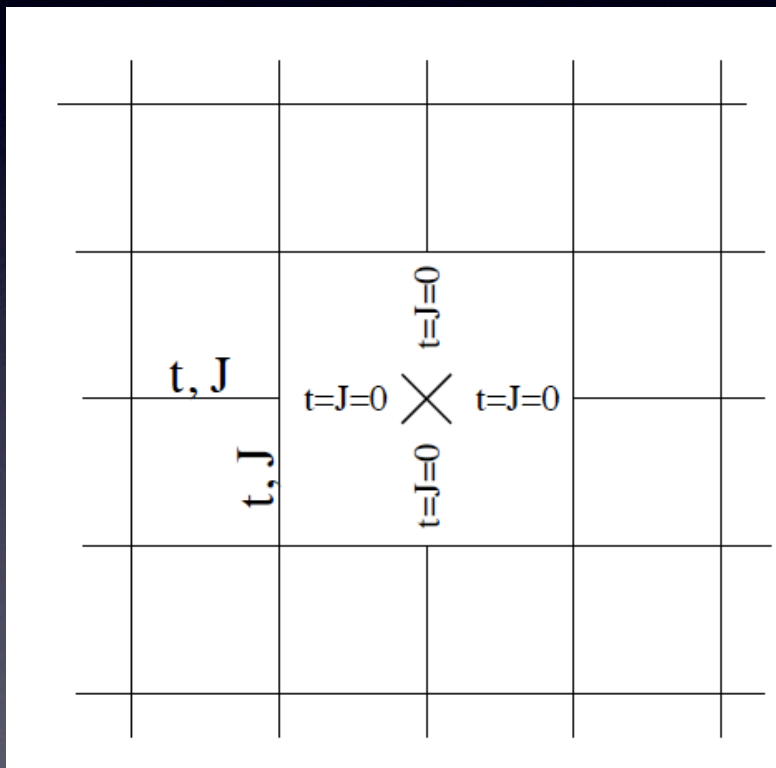
Pan et al., Nature 403, 746 (2000)



$\text{Bi}_2\text{Sr}_2\text{Ca}(\text{Cu}_{1-x}\text{Zn}_x)_2\text{O}_{8+\delta}$  single crystals



Controlled impurity doping offers a stringent test  
for correlated models !

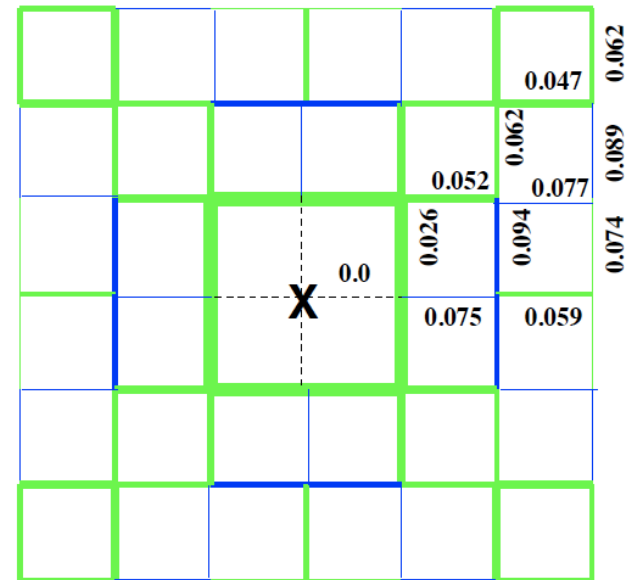


+

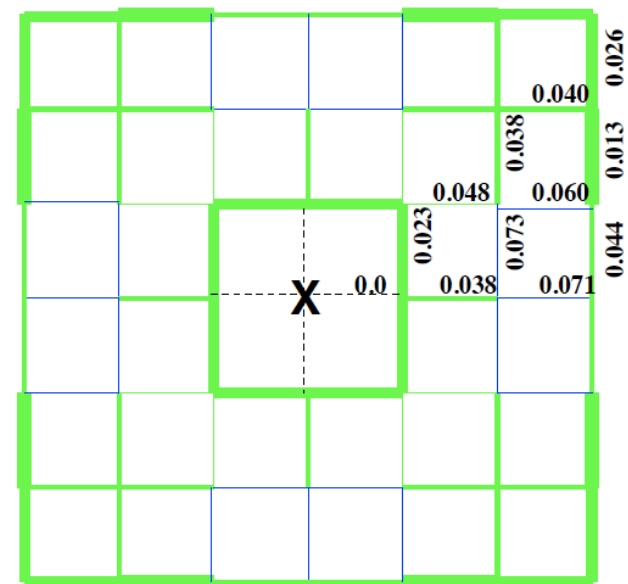
Variational Gutzwiller-  
projected RVB wavefunction  
(16x16 clusters)

Suppression of pairing correlations over large distances

$x=12\%$



$x=7\%$



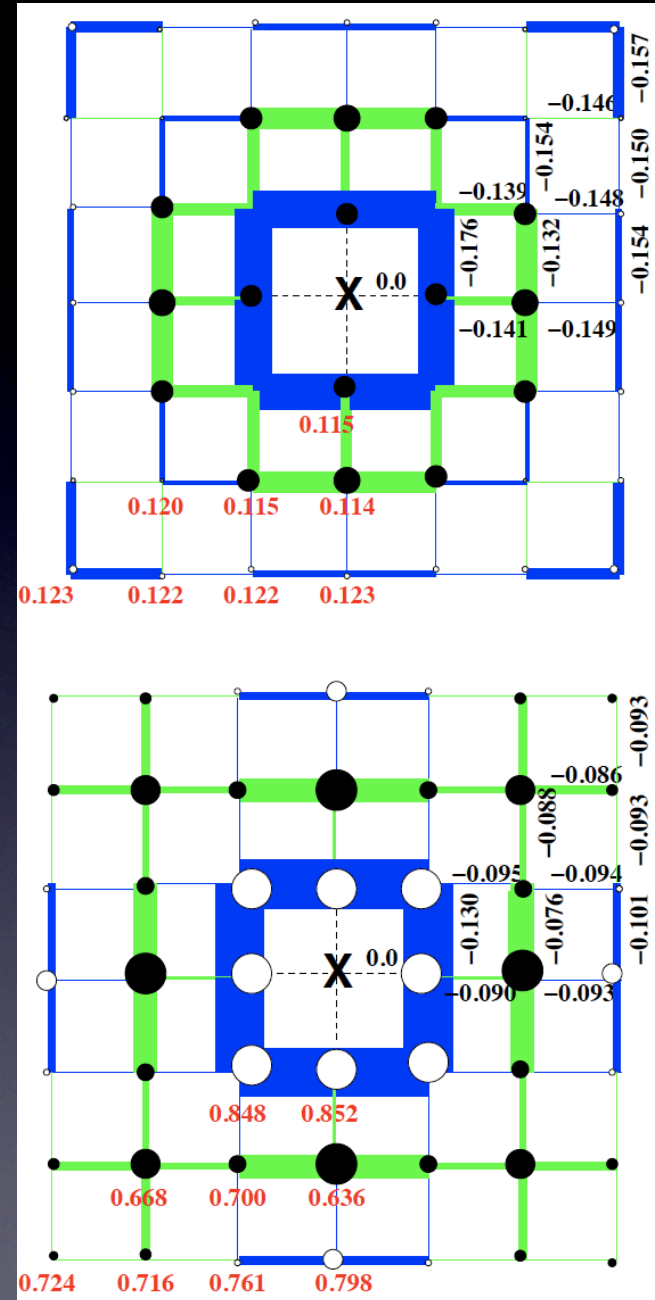
Strong modulation of local hole density

$x=12\%$

But needs STM R-maps to compare to experiments !!

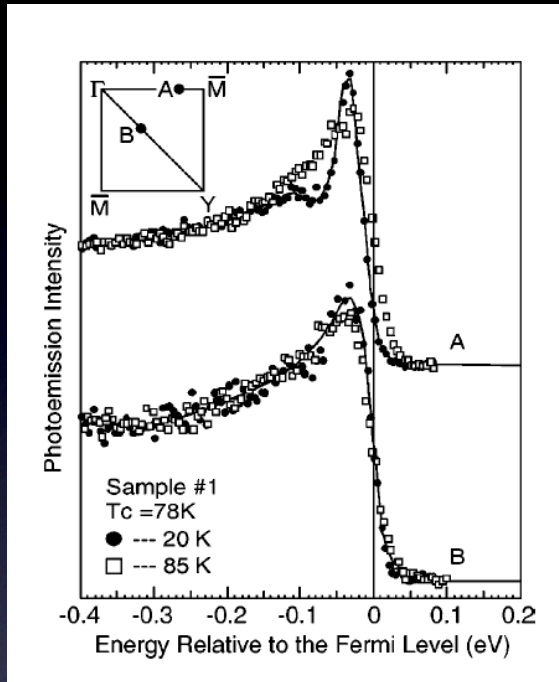
$x=7\%$

Open issue: magnetism around Zn? To compare to NMR ...

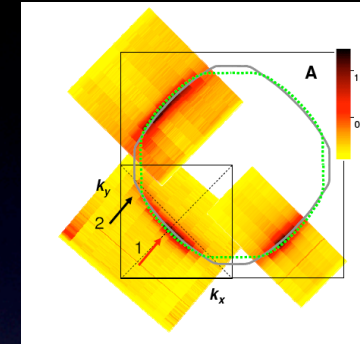




# ARPES: the d-wave gap

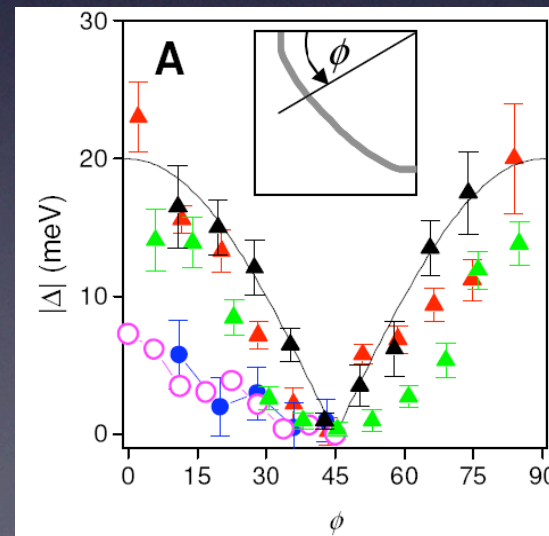


( $\pi, 0$ ) vs ( $\pi/2, \pi/2$ )  
Shen 1993  
Bi2212  
T<sub>c</sub>=88K

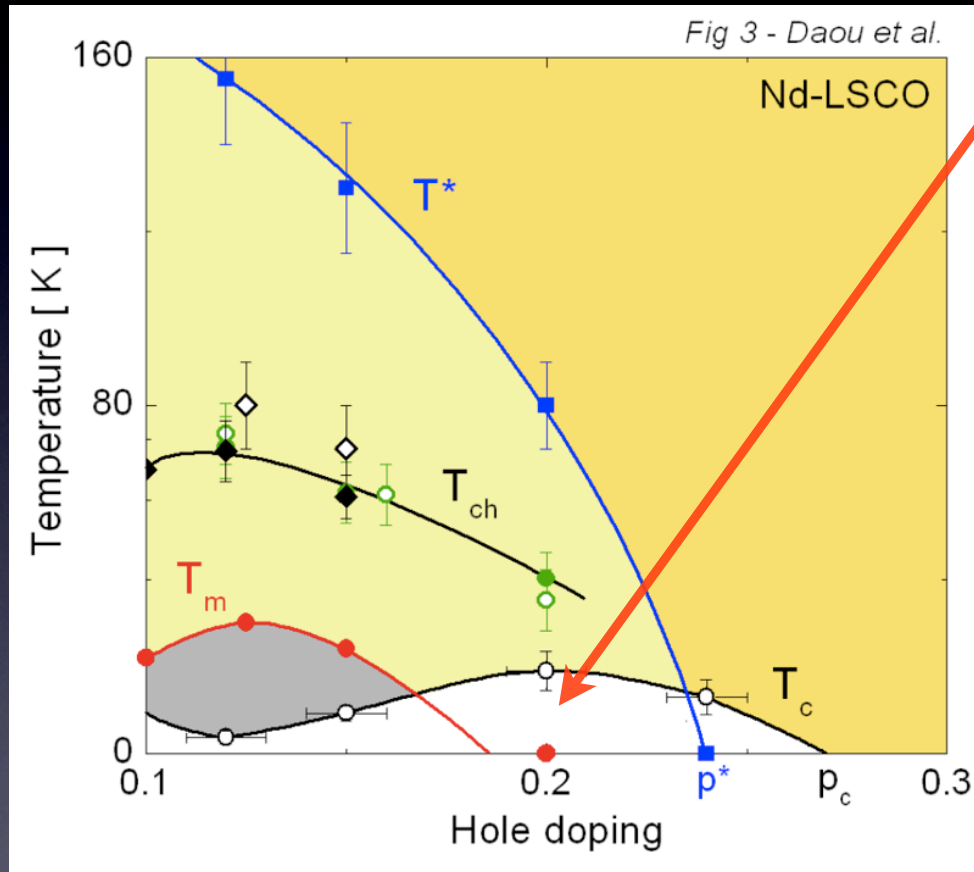


Nodal quasiparticles  
at  $\pi/2, \pi/2$

The gap closes  
at  $\pi/2, \pi/2$



Charge and Superconductivity might coexist even without spin order !



L. Taillefer's group  
[arXiv:0806.2881]

# Meanfield Fermionic theory

## Extend RVB picture & formalism to inhomogeneous case

$$\begin{aligned} H_{\text{MF}} = & -t \sum_{\langle ij \rangle \sigma} g_{ij}^t (c_{i,\sigma}^\dagger c_{j,\sigma} + h.c.) - \mu \sum_{i\sigma} n_{i,\sigma} \\ & - \frac{3}{4} J \sum_{\langle ij \rangle \sigma} g_{i,j}^J (\chi_{ji} c_{i,\sigma}^\dagger c_{j,\sigma} + h.c. - |\chi_{ij}|^2) \\ & - \frac{3}{4} J \sum_{\langle ij \rangle \sigma} g_{i,j}^J (\Delta_{ji} c_{i,\sigma}^\dagger c_{j,-\sigma}^\dagger + h.c. - |\Delta_{ij}|^2), \end{aligned}$$

- + usual MF self-consistent equations
- Site dependent g's, bond amplitudes and site densities



# RVB theory: mathematical framework

Correlated wavefunctions  
Gutzwiller projected HF d-wave BCS:

$$P|\Phi\rangle = P \prod_{\vec{k}} \left( u_{\vec{k}} + v_{\vec{k}} c_{\vec{k}\uparrow}^\dagger c_{-\vec{k}\downarrow}^\dagger \right) |0\rangle$$

$$P = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})$$

→ Variational Monte Carlo

→ Mean field theory

F.C. Zhang et al., Supercond. Sci. Technol. **1**, 36 (1988).

Gutzwiller approximation

$$\langle c_{i\sigma}^\dagger c_{j\sigma} \rangle = g_t \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle_0$$

$$\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle = g_S \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle_0$$

↓

$$H_{eff} = g_t T + g_S J \sum \mathbf{S}_i \cdot \mathbf{S}_j$$

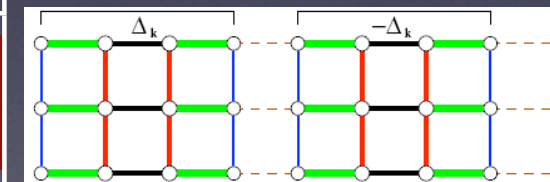
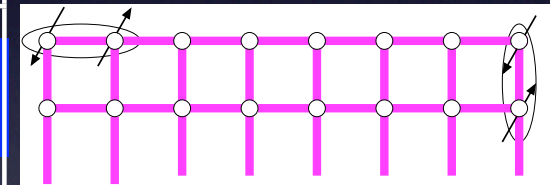
Competing phases:  
d-wave RVB  $\leftrightarrow$  staggered flux  
Affleck-Marston 1988

# Energetics for the t-J model

$$H_{tJ} = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + h.c. + J \sum_{\langle ij \rangle} S_i \cdot S_j$$

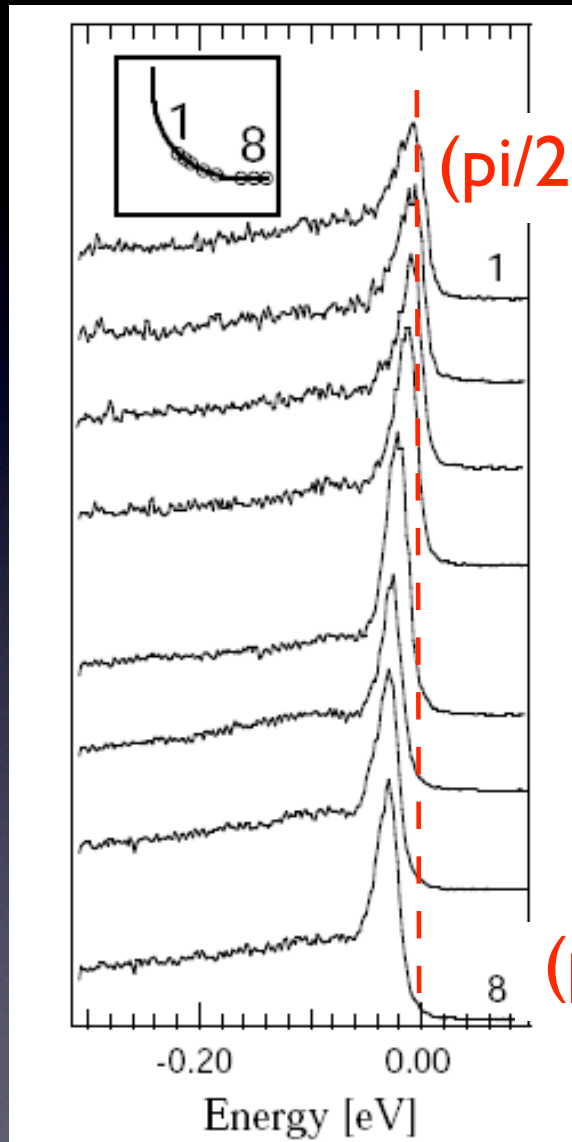
$t/J=3$ ,  
doping 1/8  
up to  
16x16 clusters

WF	$E_{\text{RMFT}} [t]$	$E_{\text{VMC}} [t]$
RVB	-0.4549	-0.45564
SFP	-0.4284	-0.44630
pi-DRVB	-0.4412	-0.44529



Very close energies but pi-shift in  $\Delta_k$  has a cost

# The d-wave superconducting gap

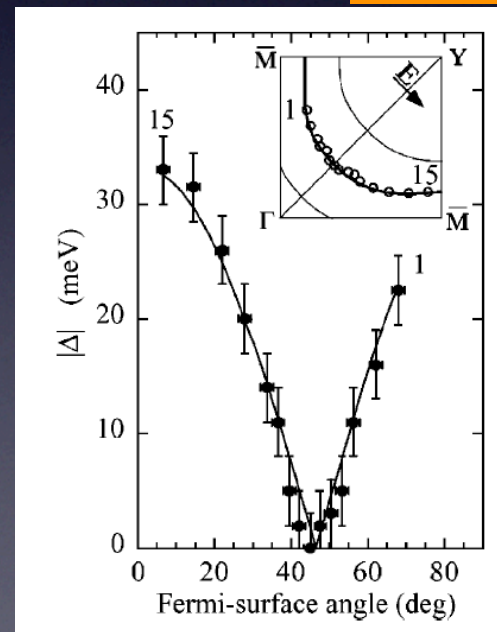


$(\pi/2, \pi/2)$

$(\pi, 0)$

Defined quasiparticles  
in the superconducting state

d-wave gap:  
the gap closes  
at  $(\pi/2, \pi/2)$

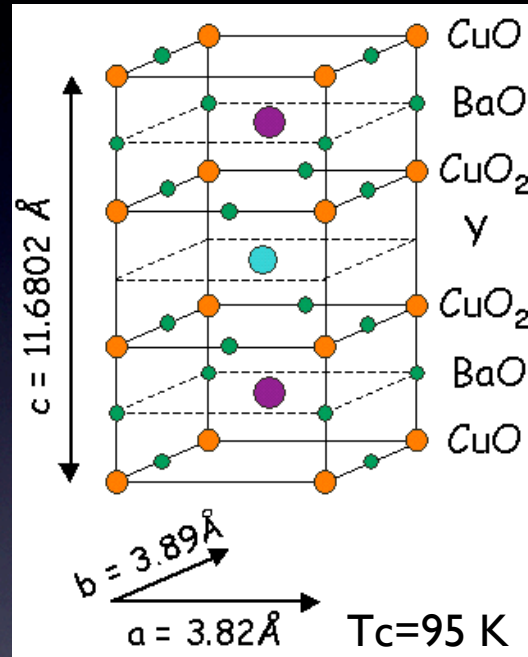
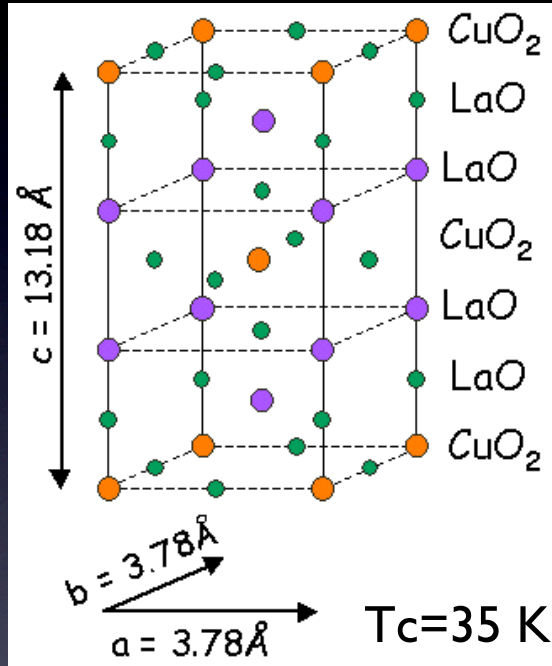


[Bi2212, Kaminski PRL 2001]

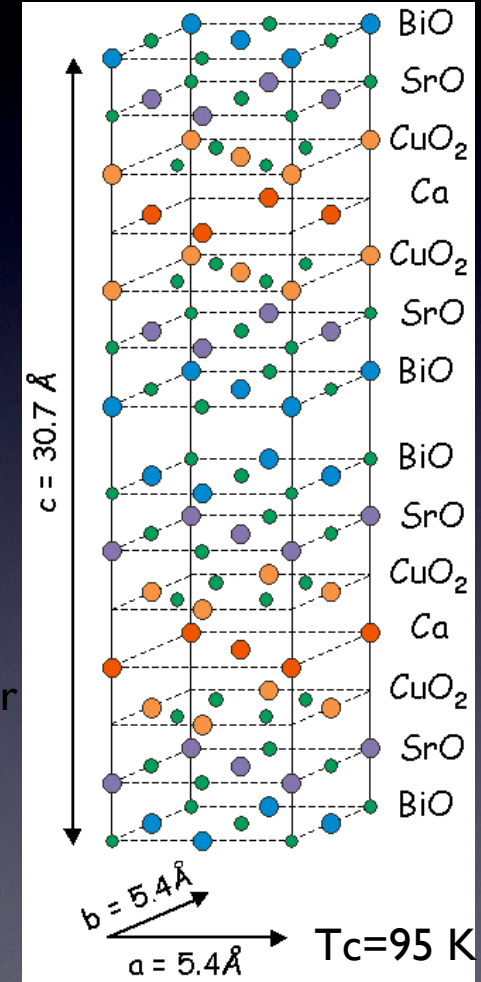
[Bi2212, Ding, Norman 1996]



# Cuprates Structure



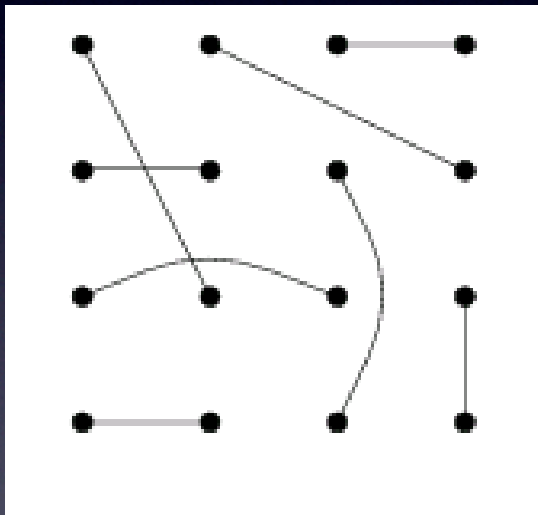
chains charge reservoir



Layered structure with  $\text{CuO}_2$  planes  
+  
charge reservoirs (La, Y, Ba, Ca, O)

# Resonating Valence Bond state

- \* Mott physics: no double occupancies
- \* Antiferromagnetic term important



Non-magnetic ground state:  
good for low spin,  
low dimensionality

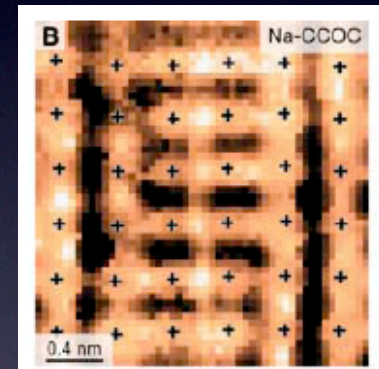
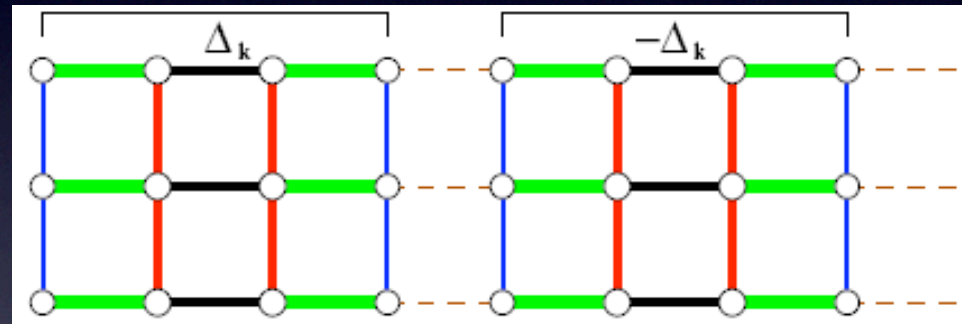
$$\bullet\text{---}\bullet = \frac{1}{\sqrt{2}} (\uparrow_i \downarrow_j - \downarrow_i \uparrow_j)$$

[Anderson, Science 1987]

RVB: liquid of singlets of spins which resonate

# Superconducting stripes (I): pi-domain RVB stripes

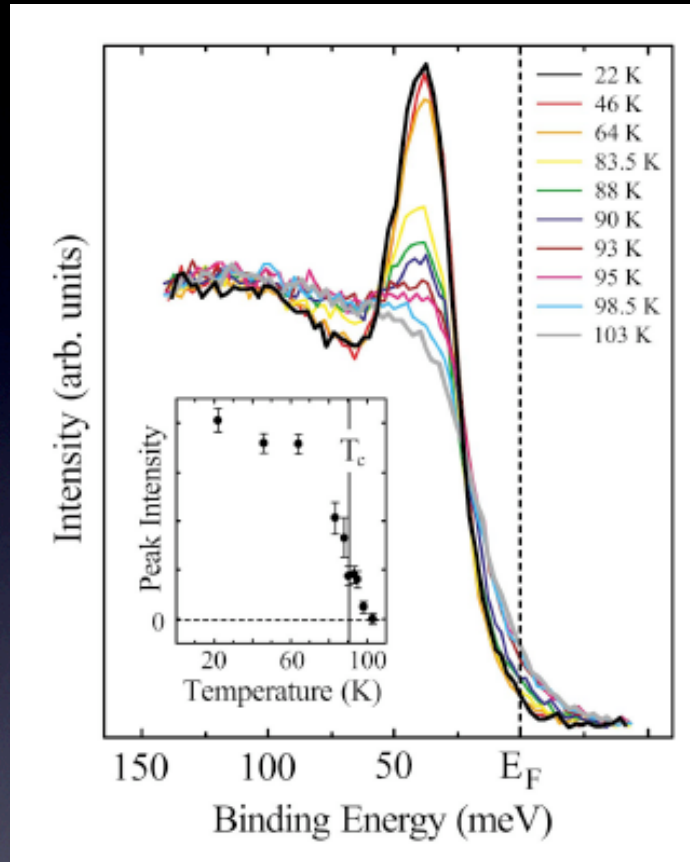
Bond-centered symmetry



The pi-phase shift in  $\Delta_k$  implies regions with domain walls in the pairing, with  $\Delta_{ij}=0$



# The pseudogap phase



$(\pi, 0)$  vs. T  
Fedorov 1999  
Bi2212  
 $T_c = 91$  K

Existence of a pseudo-gap but  
no superconductivity  
and no quasiparticles

